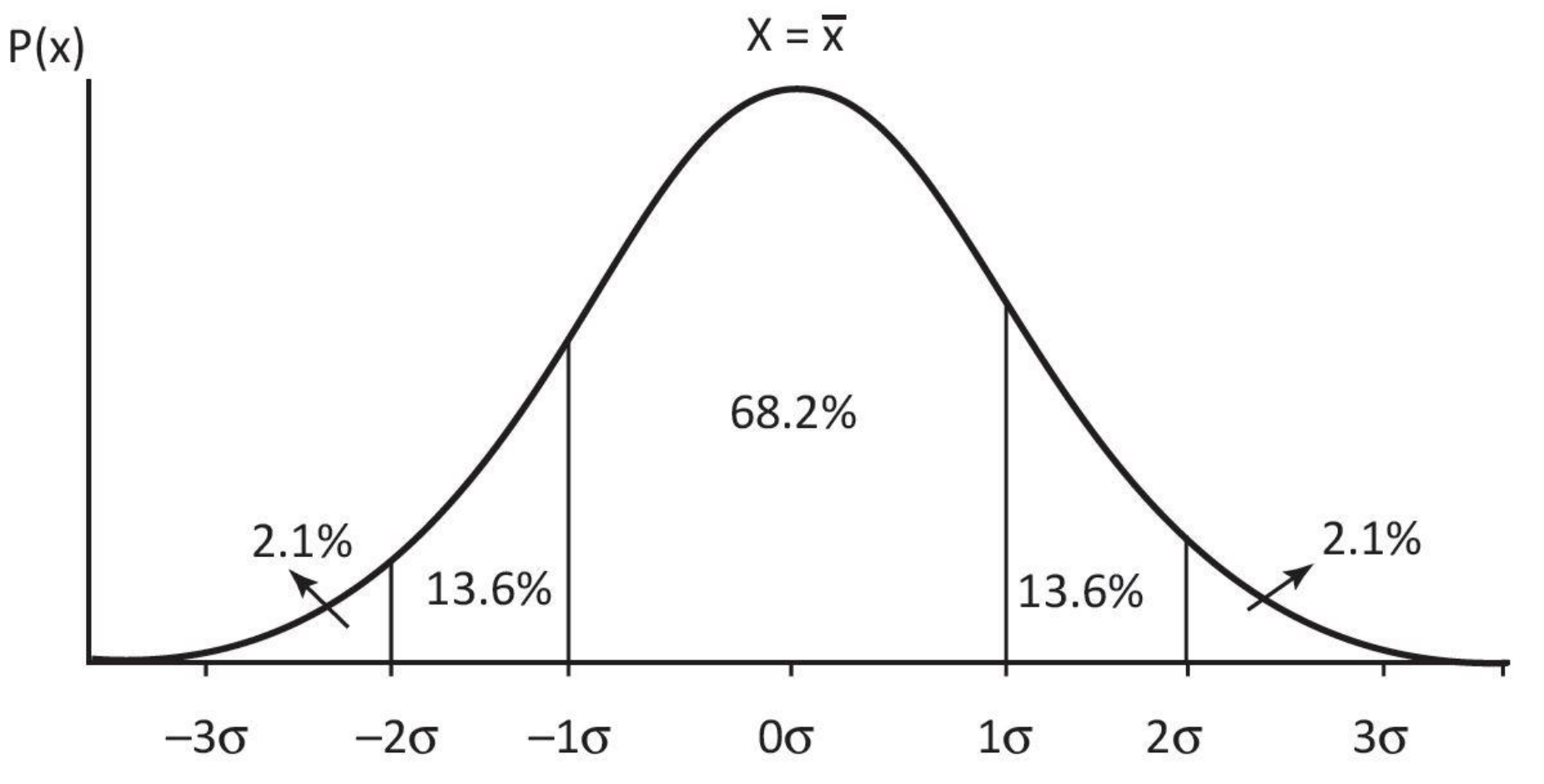
Lesson 4

Kernel Functions, Moments, Welch Test and ANOVA

Kernel function

- A function which is a central or key part of another function, and given by
- $K^*(u) = \lambda.K(\lambda.u)$,
- For example, Gaussian kernel function is the key part of the normal PDF [Equation (6.5)] in which $\lambda.\ u = [(x - \bar{x})^2 / 2\sigma^2)]$ and $\lambda = 1/\{\sigma\sqrt{(2\pi)}\}$.

Gaussian Kernel



Examples of Kernel Functions

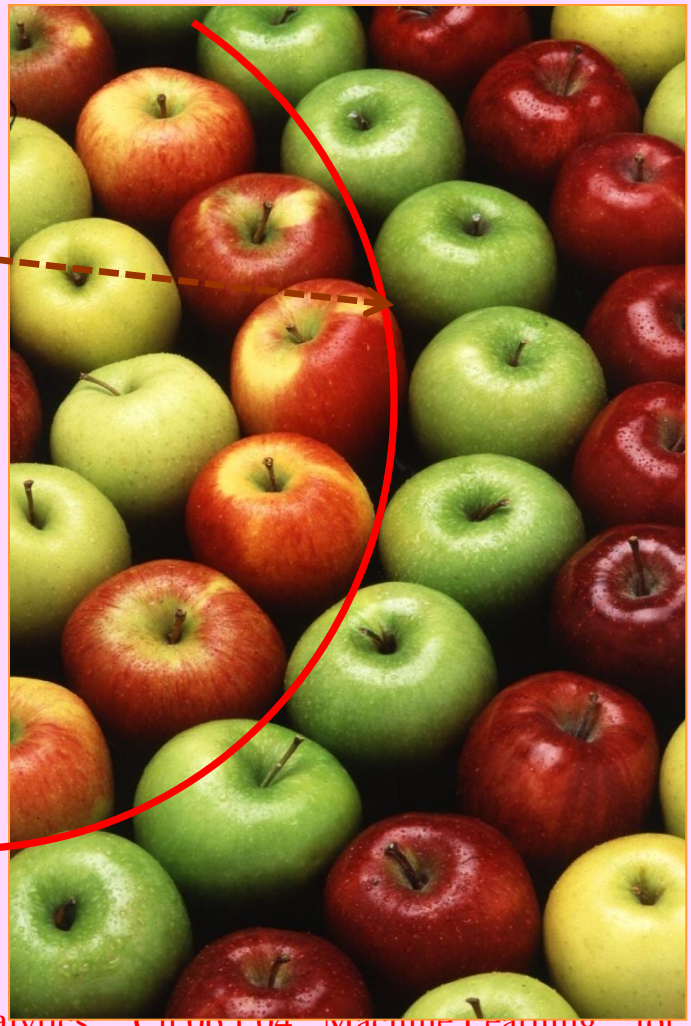
- Gaussian (equation 6.6b), Tricube (equation 6.6c), uniform, triangle, parabolic (Epanechnikov), quadratic (biweight), triweight,, quadratic and cosine.

Quadratic Kernel Function

- $K(x, y)=(x^T * y+c)^2$
- $K(x_i, x_j) = \text{Sqrt}(||x_i - x_j||^2 + c^2)$ [Multi-Quadratic] T is transpose (Column vector to row vector)
- For example, circle, parabola, ellipse, ellipsoid
- Applications: Support Vectors

Quadratic Kernel Function Application as Support Vectors to classify the Apple classes

Support Hyperplane for SVM



Cosine Kernel Function

- K(**xi**, **xj**) = **xi**.**xj**/{||**x**i||.||**x**j||}
- Vector xi length represented by ||xi||
- Vector xj length represented by ||xi||

Moments

- Refers to expected values to the powers of (0, 1, 2 ...) of variance of random variable
- 0^{th} moment is sum of $P(x) \times V[(x_i - \bar{x})^0] = 1$; 1st moment, $E(x) = \bar{x}$ = sum of $P(x) \times V[(x_i - \bar{x})^1] = \bar{x}$ (expected value) due to Gaussian kernel symmetry around $\bar{x}$

2nd,… rth Moments

- 2nd moment is squared $V[(x_i - \bar{x})^2]$ = sum of product of $[(x_i - \bar{x})^2]$, and $P(x = x_i)$.
- The r^{th} moment is sum of r^{th} power of variance $V[(x_i - \bar{x})^r]$ = sum of product of $[(x_i - \bar{x})^r]$, and $P(x = x_i)$.

Moments

- 1st moment assigns equal weight to variances of outliers and inliers
- 2nd moment assigns higher weight to outliers (distances more than σ from the mean value $\bar{x}$) compared to inliers (distance within σ from the $\bar{x}$)

Moments

- 3rd moment assigns greater weight to far outliers (distant more than 2σ from the mean value) compared to inliers,
- and so on.

Unequal Variance Welch's t-test

- A test of unequal variance, t-test
- Assumes that two groups of data are sampled data which consist of Gaussian distributed populations (Equation (6.3))
- Assume unequal σ (standard deviation)

Unequal variances *Welch's t-test*

- A two-sample location test
- Tests the hypothesis that two populations have equal means.

Hypothesis

- Making assumption statements about certain characteristics of the population
- For example, an assumption that most students of a specific professor will excel as a programmer
- Hypothesis is tested and may result in null or finite result

Welch's t-test

- An adaptation of student's t-test in statistics
- The t-test more reliable when the two samples have unequal variances and unequal sample sizes.

Analysis of Variance (ANOVA)

- Disproving or accepting the null hypothesis
- Also finds whether to accept another alternate hypothesis
- Finds that whether testing groups have any difference between them or not

Summary

We learnt:

- Kernel function
- 0^{th}, 1^{st}, 2^{nd}, …r^{th} Moments of product of PDF and the variance
- Welch t-test
- ANOVA

End of Lesson 4 on Kernel Functions, Moments, Welch Test and ANOVA